University of New Brunswick
Faculty of Computer Science
CS1303: Discrete Structures
Homework Assignment 2, ***Due Time, Date*** *11:59 PM, February 16, 2021*

Student Name: ____________________________ Matriculation Number: ________________

Instructor: Rongxing Lu
The marking scheme is shown in the left margin and [100] constitutes full marks.

[**40**] 1. For the following 4 statements, please (i) use the logical equivalences $p \to q \equiv \neg p \vee q$ and $p \leftrightarrow q \equiv (\neg p \vee q) \wedge (\neg q \vee p)$ to rewrite them without using the symbol $\to$ or $\leftrightarrow$; and (ii) use the logical equivalence $p \vee q \equiv \neg(\neg p \wedge \neg q)$ to rewrite each statement form using only $\wedge$ and $\neg$.

(a) $p \wedge \neg q \to r$

(b) $p \vee \neg q \to r \vee q$

(c) $(p \to r) \leftrightarrow (q \to r)$

(d) $(p \to (q \to r)) \leftrightarrow ((p \wedge q) \to r)$

[**20**] 2. Use truth tables to determine whether the following 4 argument forms are valid or not. Indicate which columns represent the premises and which represent the conclusion, and include a sentence explaining how the truth table supports your answer.

(a)

$$\begin{array}{ll} & p \to q \\ & q \to p \\ \therefore & p \vee q \end{array}$$

(b)

$$\begin{array}{ll} & p \\ & p \to q \\ & \neg q \vee r \\ \therefore & r \end{array}$$

(c)

$$\begin{array}{ll} & p \vee q \\ & p \to \neg q \\ & p \to r \\ \therefore & r \end{array}$$

(d)

$$\begin{array}{ll} & p \wedge q \to \neg r \\ & p \vee \neg q \\ & \neg q \to p \\ \therefore & \neg r \end{array}$$

[20] 3. A set of premises and a conclusion are given. Use the valid argument forms listed in Table 1 to deduce the conclusion from the premises, giving a reason for each step. Assume all variables are statement variables.

(a)

a.		$\neg p \vee q \to r$
b.		$s \vee \neg q$
c.		$\neg t$
d.		$p \to t$
e.		$\neg p \wedge r \to \neg s$
f.	$\therefore$	$\neg q$

(b)

a.		$p \vee q$
b.		$q \to r$
c.		$p \wedge s \to t$
d.		$\neg r$
e.		$\neg q \to u \wedge s$
f.	$\therefore$	t

Modus Ponens	$p \to q$ p $\therefore q$	**Elimination**	**a.** $p \vee q$ $\sim q$ $\therefore p$ **b.** $p \vee q$ $\sim p$ $\therefore q$
Modus Tollens	$p \to q$ $\sim q$ $\therefore \sim p$	**Transitivity**	$p \to q$ $q \to r$ $\therefore p \to r$
Generalization	**a.** p $\therefore p \vee q$ **b.** q $\therefore p \vee q$	**Proof by Division into Cases**	$p \vee q$ $p \to r$ $q \to r$ $\therefore r$
Specialization	**a.** $p \wedge q$ $\therefore p$ **b.** $p \wedge q$ $\therefore q$		
Conjunction	p q $\therefore p \wedge q$	**Contradiction Rule**	$\sim p \to \mathbf{c}$ $\therefore p$

Table 1: Valid Arguments

[20] 4. The logician Raymond Smullyan describes an island containing two types of people: knights who always tell the truth and knaves who always lie. Now, you are visiting the island and have the following encounters with natives.

(a) Two natives A and B address you as follows:
A says: Both of us are knights. B says: A is a knave. What are A and B?

(b) Another two natives C and D approach you but only C speaks.
C says: Both of us are knaves. What are C and D?

(c) You then encounter natives E and F.
E says: F is a knave. F says: E is a knave. How many knaves are there?