University of New Brunswick
Faculty of Computer Science
CS1303: Discrete Structures
Homework Assignment 4, ***Due Time, Date*** *11:59 PM, March 9, 2021*

Student Name: _____ Matriculation Number: _____

Instructor: Rongxing Lu
The marking scheme is shown in the left margin and [100] constitutes full marks.

[**10**] 1. Write a negation for the given statement, and use a counterexample to disprove the given statement. Explain how the counterexample actually shows that the given statement is false.

(a) For all real numbers a and b, if $a < b$ then $a^2 < b^2$.

(b) For every integer n, if n is odd then $\frac{n-1}{2}$ is odd.

[**15**] 2. Disprove each of the following statements by giving a counterexample. In each case explain how the counterexample actually disproves the statement.

(a) For all integers m and n, if $2m + n$ is odd then m and n are both odd.

(b) For every integer p, if p is prime then $p^2 - 1$ is even.

(c) For every integer n, if n is even then $n^2 + 1$ is prime.

[**20**] 3. Determine which of the following statements are true and which are false. Prove each true statement directly from the definitions, and give a counterexample for each false statement.

(a) The product of any two rational numbers is a rational number.

(b) The quotient of any two rational numbers is a rational number.

(c) The difference of any two rational numbers is a rational number.

(d) If r and s are any two rational numbers, then $\frac{r+s}{2}$ is rational.

[**25**] 4. Determine whether the statement is true or false. Prove the statement directly from the definitions if it is true, and give a counterexample if it is false.

(a) The sum of any three consecutive integers is divisible by 3.

(b) The product of any two even integers is a multiple of 4.

(c) For all integers a, b, and c, if $a|b$ and $a|c$ then $a|(2b - 3c)$.

(d) For all integers a, b, and c, if $ab|c$ then $a|c$ and $b|c$.

(e) For all integers a, b, and c, if $a|(b + c)$ then $a|b$ or $a|c$.

[**30**] 5. Prove each of the following statements by contradiction. In addition, for (e) and (f), please also prove them by contraposition.

(a) For all odd integers a and b, $b^2 - a^2 \neq 4$.

(b) For all prime numbers a, b, and c, $a^2 + b^2 \neq c^2$.

(c) If a and b are rational numbers, $b \neq 0$, and r is an irrational number, then $a + br$ is irrational.

(d) For any integer n, $n^2 - 2$ is not divisible by 4.

(e) The negative of any irrational number is irrational.

(f) For every integer n, if n^2 is odd then n is odd.