University of New Brunswick
Faculty of Computer Science
CS1303: Discrete Structures
Homework Assignment 5, **Due Time, Date** *11:59 PM, March 23, 2021*

Student Name: ____________________________ Matriculation Number: ________________

Instructor: Rongxing Lu
The marking scheme is shown in the left margin and [100] constitutes full marks.

[**5**] 1. Transform each of the following by making the change of variable $j = i - 1$.

(a)
$$\sum_{i=1}^{n+1} \frac{(i-1)^2}{i \cdot n}$$

(b)
$$\sum_{i=3}^{n} \frac{i}{i+n-1}$$

[**5**] 2. Write each of following as a single summation or product.

(a)
$$3 \cdot \sum_{k=1}^{n} (2k-3) + \sum_{k=1}^{n} (4-5k)$$

(b)
$$\left(\prod_{k=1}^{n} \frac{k}{k+1} \right) \cdot \left(\prod_{k=1}^{n} \frac{k+1}{k+2} \right)$$

[**10**] 3. Compute each of the following. Assume the values of the variables are restricted so that the expressions are defined.

(a)
$$\frac{4!}{3!}$$

(b)
$$\frac{3!}{0!}$$

(c)
$$\frac{(n-1)!}{(n+1)!}$$

(d)
$$\frac{n!}{(n-k+1)!}$$

[**28**] 4. Prove each of the following statements using mathematical induction.

(a) For every integer $n \geq 1$,

$$1 + 6 + 11 + 16 + \cdots + (5n - 4) = \frac{n(5n-3)}{2}.$$

(b) For every integer $n \geq 3$,

$$4^3 + 4^4 + 4^5 + \cdots + 4^n = \frac{4(4^n - 16)}{3}.$$

(c) For every integer $n \geq 1$,

$$1^2 + 2^2 + \cdots + n^2 = \frac{n(n+1)(2n+1)}{6}.$$

(d) For every integer $n \geq 1$,

$$\sum_{i=1}^{n} i(i!) = (n+1)! - 1.$$

[**32**] 5. Prove each of the following statements using mathematical induction.

(a) For each integer $n \geq 0$, $3^{2n} - 1$ is divisible by 8.

(b) For each integer $n \geq 2$, $2^n < (n+1)!$.

(c) For each integer $n \geq 0$, $1 + 3n \leq 4^n$.

(d) For every real number $x > -1$ and every integer $n \geq 2$, $1 + nx \leq (1+x)^n$.

[**20**] 6. Prove each of the following statements using strong mathematical induction.

(a) Suppose $b_1, b_2, b_3, \cdots$ is a sequence defined as follows:

$$b_1 = 4, \quad b_2 = 12, \qquad b_k = b_{k-2} + b_{k-1} \qquad \text{for each integer } k \geq 3$$

Prove that b_n is divisible by 4 for every integer $n \geq 1$.

(b) Suppose $f_0, f_1, f_2, \cdots$ is a sequence defined as follows:

$$f_0 = 5, \quad f_1 = 16, \qquad f_k = 7f_{k-1} - 10f_{k-2} \qquad \text{for each integer } k \geq 2$$

Prove that $f_n = 3 \cdot 2^n + 2 \cdot 5^n$ for every integer $n \geq 0$.