University of New Brunswick
Faculty of Computer Science
CS1303: Discrete Structures
Homework Assignment 6, ***Due Time, Date*** *11:59 PM, April 2, 2021*

Student Name: _____ Matriculation Number: _____

Instructor: Rongxing Lu
The marking scheme is shown in the left margin and [100] constitutes full marks.

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[16] 1. Let $A = \{1, 3, 5, 7, 9\}$, $B = \{3, 6, 9\}$, and $C = \{2, 4, 6, 8\}$. Find each of the following:

(a) $A \cup B$
(b) $A \cap B$
(c) $A \cup C$
(d) $A \cap C$
(e) $A - B$
(f) $B - A$
(g) $B \cup C$
(h) $B \cap C$

[8] 2. Let S be the set of all strings of 0's and 1's of length 4, and let A and B be the following subsets of S: $A = \{1110, 1111, 1000, 1001\}$ and $B = \{1100, 0100, 1111, 0111\}$. Find each of the following:

(a) $A \cup B$
(b) $A \cap B$
(c) $A - B$
(d) $B - A$

[20] 3. In each of the following, draw a Venn diagram for sets A, B, and C that satisfy the given conditions.

(a) $A \subseteq B$, $C \subseteq B$, $A \cap C = \varnothing$
(b) $B \subseteq A$, $B \cap C = \varnothing$
(c) $A \cap B = \varnothing$, $A \subseteq C$, $C \cap B \neq \varnothing$
(d) $A \cap B \neq \varnothing$, $B \cap C \neq \varnothing$, $A \cap C = \varnothing$, $A \not\subseteq B$, $C \not\subseteq B$

[16] 4. Let $A = \{a, b\}$, $B = \{1, 2\}$, $C = \{2, 3\}$. Find each of the following sets.

(a) $A \times (B \cup C)$
(b) $(A \times B) \cup (A \times C)$
(c) $A \times (B \cap C)$

(d) $(A \times B) \cap (A \times C)$

[**10**] 5. Let Z be the set of all integers and let

$$A_0 = \{n \in Z | n = 4k + 0, k \in Z\}$$

$$A_1 = \{n \in Z | n = 4k + 1, k \in Z\}$$

$$A_2 = \{n \in Z | n = 4k + 2, k \in Z\}$$

$$A_3 = \{n \in Z | n = 4k + 3, k \in Z\}$$

Is (A_0, A_1, A_2, A_3) a partition of Z? Explain your answer.

[**20**] 6. Assume that all sets are subsets of a universal set U. Please prove each statement below.

(a) For all sets A, B, and C, if $B \cap C \subseteq A$, then $(C - A) \cap (B - A) = \varnothing$.

(b) For all sets A, B, C, and D, if $A \cap C = \varnothing$, then $(A \times B) \cap (C \times D) = \varnothing$.

(c) For every positive integer n, if A and $B_1, B_2, B_3, \cdots$ are any sets, then

$$A \cap \left(\bigcup_{i=1}^{n} B_i \right) = \bigcup_{i=1}^{n} (A \cap B_i)$$

(d) For every positive integer n, if A and $B_1, B_2, B_3, \cdots$ are any sets, then

$$\bigcup_{i=1}^{n} (A \times B_i) = A \times \left(\bigcup_{i=1}^{n} B_i \right)$$

[**10**] 7. Find a counterexample to show that the each statement is false. Assume all sets are subsets of a universal set U.

(a) For all sets A, B, and C,

$$(A \cup B) \cap C = A \cup (B \cap C).$$

(b) For all sets A, B, and C, if $A \not\subseteq B$ and $B \not\subseteq C$ then $A \not\subseteq C$.