University of New Brunswick
Faculty of Computer Science
CS1303: Discrete Structures
Homework Assignment 3, ***Due Time, Date*** *11:59 PM, February 23, 2021*

Student Name: _____ Matriculation Number: _____

Instructor: Rongxing Lu
The marking scheme is shown in the left margin and [100] constitutes full marks.

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[20] 1. Let $D = \{-48, -14, -8, 0, 1, 3, 16, 23, 26, 32, 36\}$. Determine which of the following statements are true and which are false. Provide counterexamples for the statements that are false.

(a) $\forall x \in D$, if x is odd then $x > 0$.
(b) $\forall x \in D$, if x is less than 0 then x is even.
(c) $\forall x \in D$, if x is even then $x \leq 0$.
(d) $\forall x \in D$, if the ones digit of x is 2, then the tens digit is 3 or 4.
(e) $\forall x \in D$, if the ones digit of x is 6, then the tens digit is 1 or 2.

[10] 2. Let $D = E = \{-2, -1, 0, 1, 2\}$. Explain why the following statements are true.

(a) $\forall x$ in D, $\exists y$ in E such that $x + y = 0$.
(b) $\exists x$ in D such that $\forall y$ in E, $x + y = y$.

[20] 3. Please rewrite the following statements formally using quantifiers and variables, and write a negation for each statement.

(a) Everybody loves somebody.
(b) Somebody loves everybody.
(c) Any even integer equals twice some integer.
(d) Every action has an equal and opposite reaction.
(e) There is a program that gives the correct answer to every question that is posed to it.

[50] 4. Some of the following arguments are valid by universal modus ponens or universal modus tollens; others are invalid. State which are valid and which are invalid. Justify your answers.

(a)

All healthy people eat an apple a day.
Alice eats an apple a day.
$\therefore$ Alice is a healthy person.

(b)

For every student x, if x studies discrete mathematics, then x is good at logic.
Bob studies discrete mathematics.
$\therefore$ Bob is good at logic.

(c)

If compilation of a computer program produces error messages, then the program is not correct.
Compilation of this program does not produce error messages.

$\therefore$ This program is correct.

(d)

Any product of two positive numbers is positive.
The product $p \cdot q$ is positive.

$\therefore$ The numbers p and q are both positive.

(e)

If a number is even, then twice that number is even.
The number $2n$ is even, for a particular number n.

$\therefore$ The particular number n is even.