University of New Brunswick
Faculty of Computer Science

CS1303 Discrete Structures - Quiz 1

January 29th, 2021;
Time Allowed: 20 miniutes

Student Name: ______________________ **Student No.:** ____________________

Instructions
This paper contains 5 multiple choice questions and 1 proof question, and comprises 2 pages.
Answer ALL questions. This is an open-book examination.
The marking scheme is shown in the left margin and [100] constitutes full marks.

[**75**] 1. **Multiple choice questions:** read each question carefully and choose the correct answer: A, B, C or D. Make sure you only choose one answer for each question.

[15] (1) Find which of the following sentences is a proposition: ________.

A. CS1303 is very easy, isn't it?

B. Drink carrot juice!

C. Is there life of Mars?

D. Java language belongs to high-level programming languages.

[15] (2) Let $A = \{a, b, c\}$. If we list all the strings of length 3 over A with at least two characters that are the same into a string set B. What is the size of set B ________.

A. 27 B. 8 C. 21 D. 18

[15] (3) Which of the following statements is false? ________

A. $3 \in \{1, 2, 3\}$ B. $\{3\} \in \{1, 2, \{3\}\}$

C. $\{3\} \subseteq \{1, 2, \{3\}\}$ D. $\{1\} \subseteq \{1, 2, \{3\}\}$

[15] (4) Let $\oplus$ denote as the *exclusive or* operation. Which of the following compound propositions is a tautology? ________.

A. $p \oplus p$ B. $p \oplus \neg p$ C. $p \oplus \neg q$ D. $\neg p \oplus \neg q$

[15] (5) Which of the following compound propositions is logically equivalent to $(p \vee q) \wedge \neg(p \wedge q)$? ________.

A. $(p \wedge \neg q) \vee (\neg p \wedge q)$ B. $\neg(p \wedge q) \vee (\neg p \wedge \neg q)$

C. $\neg((p \wedge q) \wedge (\neg p \wedge \neg q))$ D. $(p \vee q) \vee \neg(p \wedge q)$

[**25**] 2. Show that

$$(p \to r) \land (q \to r) \equiv (p \lor q) \to r$$

by

(a) Using truth tables;

(b) Using logical equivalences.

END OF PAPER

The following table may be needed for taking this exam.

Given any statement variables p, q, and r, a tautology $\mathbf{t}$ and a contradiction $\mathbf{c}$, the following logical equivalences hold.

1. *Commutative laws:*	$p \land q \equiv q \land p$	$p \lor q \equiv q \lor p$
2. *Associative laws:*	$(p \land q) \land r \equiv p \land (q \land r)$	$(p \lor q) \lor r \equiv p \lor (q \lor r)$
3. *Distributive laws:*	$p \land (q \lor r) \equiv (p \land q) \lor (p \land r)$	$p \lor (q \land r) \equiv (p \lor q) \land (p \lor r)$
4. *Identity laws:*	$p \land \mathbf{t} \equiv p$	$p \lor \mathbf{c} \equiv p$
5. *Negation laws:*	$p \lor \sim p \equiv \mathbf{t}$	$p \land \sim p \equiv \mathbf{c}$
6. *Double negative law:*	$\sim(\sim p) \equiv p$	
7. *Idempotent laws:*	$p \land p \equiv p$	$p \lor p \equiv p$
8. *Universal bound laws:*	$p \lor \mathbf{t} \equiv \mathbf{t}$	$p \land \mathbf{c} \equiv \mathbf{c}$
9. *De Morgan's laws:*	$\sim(p \land q) \equiv \sim p \lor \sim q$	$\sim(p \lor q) \equiv \sim p \land \sim q$
10. *Absorption laws:*	$p \lor (p \land q) \equiv p$	$p \land (p \lor q) \equiv p$
11. *Negations of* **t** *and* **c**:	$\sim\mathbf{t} \equiv \mathbf{c}$	$\sim\mathbf{c} \equiv \mathbf{t}$

In addition, Representation of If-Then as Or (Implication Equivalence) means $p \to q \equiv \neg p \lor q$, which may also be used.

Solutions.

1.

D, C, C, B, A

2.a

p	q	r	$p \to r$	$q \to r$	$(p \to r) \land (q \to r)$	$p \lor q$	$(p \lor q) \to r$
T	T	T	T	T	T	T	T
T	F	T	T	T	T	T	T
F	T	T	T	T	T	T	T
F	F	T	T	T	T	F	T
T	T	F	F	F	F	T	F
T	F	F	F	T	F	T	F
F	T	F	T	F	F	T	F
F	F	F	T	T	T	F	T

2.b

$$\begin{aligned} (p \to r) \land (q \to r) &\equiv (\neg p \lor r) \land (\neg q \lor r) && \text{by Representation of If-Then as Or} \\ &\equiv (\neg p \land \neg q) \lor r && \text{by Distributive laws} \\ &\equiv \neg(p \lor q) \lor r && \text{by De Morgan's law} \\ &\equiv (p \lor q) \to r && \text{by Representation of If-Then as Or} \end{aligned}$$