University of New Brunswick
Faculty of Computer Science
CS1303: Discrete Structures
Homework Assignment 1, ***Due Time, Date*** *11:59 PM, February 2, 2021*

Student Name: ______________________________ Matriculation Number: _________________

Instructor: Rongxing Lu
The marking scheme is shown in the left margin and [100] constitutes full marks.

[**10**] 1. Fill in the blanks to rewrite the given statements below.

(a) For every object J, if J is a square then J has four sides.

i. All squares ______________________.
ii. Every square ______________________.
iii. If an object is a square, then it ______________________.
iv. If J ______________________, then J ______________________.
v. For every square J, ______________________.

(b) Every positive number has a positive square root.

i. All positive numbers ______________________.
ii. For every positive number e, there is ______________________ for e.
iii. For every positive number e, there is a positive number r such that ______________________.

(c) There is a real number whose product with every number leaves the number unchanged.

i. There is a real number r such that the product of r ______________________.
ii. There is a real number r with the property that for every real number s, ______________________.

[**20**] 2. Let $A = \{w, x, y, z\}$, and $B = \{a, b\}$. Use the set roster notation to write each of the following sets, and indicate the number of elements that are in each set.

(a) $A \times B$.
(b) $B \times A$.
(c) $A \times A$.
(d) $B \times B$.

[**10**] 3. Let $S = \{0, 1\}$. List all the strings of length 4 over S that contain three or more 0's.

[**20**] 4. Let p, q, and r be the propositions

p: You have the flu.

q: You miss the final examination.

r: You pass the course.

Express each of these propositions as an English sentence.

(a) $p \to q$.

(b) $q \to \neg r$.

(c) $p \vee q \vee r$.

(d) $(p \to \neg r) \vee (q \to \neg r)$.

(e) $(p \wedge q) \vee (\neg q \wedge r)$.

[**20**] 5. Construct a truth table for each of these compound propositions.

(a) $p \wedge \neg p$.

(b) $p \vee \neg p$.

(c) $(p \vee \neg q) \to r$.

(d) $(p \vee q) \to (p \wedge q)$.

(e) $(p \to q) \to (q \to p)$.

[**20**] 6. Use the Laws of Equivalence to prove the following logical equivalences. State clearly which law(s) you are using in each step.

(a) $\neg(p \vee (\neg p \wedge q)) \equiv \neg p \wedge \neg q$.

(b) $(p \wedge q) \to (p \vee q) \equiv \mathbf{t}$.

(c) $p \to (q \vee r) \equiv (p \wedge \neg q) \to r$.

(d) $p \to (q \to r) \equiv (p \wedge q) \to r$.

Solutions.

[**10**] 1. Fill in the blanks to rewrite the given statements below.

(a) For every object J, if J is a square then J has four sides.

i. All squares ____________________.
✓ have four sides.

ii. Every square ____________________.
✓ has four sides.

iii. If an object is a square, then it ____________________.
✓ has four sides.

iv. If J ____________________, then J ____________________.
✓ is a square, has four sides.

v. For every square J, ____________________.
✓ J has four sides.

(b) Every positive number has a positive square root.

i. All positive numbers ____________________.
✓ have a positive square root.

ii. For every positive number e, there is ____________________ for e.
✓ a square root

iii. For every positive number e, there is a positive number r such that ____________________.
✓ $r = \sqrt{e}$

(c) There is a real number whose product with every number leaves the number unchanged.

i. There is a real number r such that the product of r ____________________.
✓ with every number leaves the number unchanged.

ii. There is a real number r with the property that for every real number s, ____________________.
✓ $r \times s = s$

[**20**] 2. Let $A = \{w, x, y, z\}$, and $B = \{a, b\}$. Use the set roster notation to write each of the following sets, and indicate the number of elements that are in each set.

(a) $A \times B$.
✓

$$A \times B = \{(w, a), (w, b), (x, a), (x, b), (y, a), (y, b), (z, a), (z, b)\}$$
$$A \times B \text{ has } 4 \cdot 2 = 8 \text{ elements.}$$

(b) $B \times A$.
✓

$$B \times A = \{(a, w), (a, x), (a, y), (a, z), (b, w), (b, x), (b, y), (b, z)\}$$
$$B \times A \text{ has } 2 \cdot 4 = 8 \text{ elements.}$$

(c) $A \times A$.

✓

$$A \times A = \{(w,w),(w,x),(w,y),(w,z),(x,w),(x,x),(x,y),(x,z),$$
$$(y,w),(y,x),(y,y),(y,z),(z,w),(z,x),(z,y),(z,z)\}$$

$A \times A$ has $4 \cdot 4 = 16$ elements.

(d) $B \times B$.

✓

$$B \times B = \{(a,a),(a,b),(b,a),(b,b)\}$$

$B \times B$ has $2 \cdot 2 = 4$ elements.

[**10**] 3. Let $S = \{0, 1\}$. List all the strings of length 4 over S that contain three or more 0's.

✓

$$1000, 0100, 0010, 0001, 0000$$

[**20**] 4. Let p, q, and r be the propositions

p: You have the flu.

q: You miss the final examination.

r: You pass the course.

Express each of these propositions as an English sentence.

(a) $p \to q$.

✓ If you have the flu, then you miss the final examination.

(b) $q \to \neg r$.

✓ If you miss the final examination, then you do not pass the course.

(c) $p \vee q \vee r$.

✓ You have the flu or you miss the final examination or you pass the course.

(d) $(p \to \neg r) \vee (q \to \neg r)$.

✓ If you have the flu, then you do not pass the course; or if you miss the final examination, then you do not pass the course.

(e) $(p \wedge q) \vee (\neg q \wedge r)$.

✓ You have the flu and you miss the final examination, or you do not miss the final examination and you pass the course.

[**20**] 5. Construct a truth table for each of these compound propositions.

(a) $p \wedge \neg p$.

p	$\neg p$	$p \wedge \neg p$
T	F	F
F	T	F

(b) $p \vee \neg p$.

p	$\neg p$	$p \vee \neg p$
T	F	T
F	T	T

(c) $(p \vee \neg q) \to r$.

p	q	r	$\neg q$	$p \vee \neg q$	$(p \vee \neg q) \to r$
T	T	T	F	T	T
T	F	T	T	T	T
F	T	T	F	F	T
F	F	T	T	T	T
T	T	F	F	T	F
T	F	F	T	T	F
F	T	F	F	F	T
F	F	F	T	T	F

(d) $(p \vee q) \to (p \wedge q)$.

p	q	$p \vee q$	$p \wedge q$	$(p \vee q) \to (p \wedge q)$
T	T	T	T	T
F	T	T	F	F
T	F	T	F	F
F	F	F	F	T

(e) $(p \to q) \to (q \to p)$.

p	q	$p \to q$	$q \to p$	$(p \to q) \to (q \to p)$
T	T	T	T	T
F	T	T	F	F
T	F	F	T	T
F	F	T	T	T

[**20**] 6. Use the Laws of Equivalence to prove the following logical equivalences. State clearly which law(s) you are using in each step.

(a) $\neg(p \vee (\neg p \wedge q)) \equiv \neg p \wedge \neg q$.

$\neg(p \vee (\neg p \wedge q))$	$\equiv$	$\neg p \wedge \neg(\neg p \wedge q)$	by De Morgan's laws
	$\equiv$	$\neg p \wedge (\neg(\neg p) \vee \neg q)$	by De Morgan's laws
	$\equiv$	$\neg p \wedge (p \vee \neg q)$	by Double negation laws
	$\equiv$	$(\neg p \wedge p) \vee (\neg p \wedge \neg q)$	by Distributive laws
	$\equiv$	$\mathbf{c} \vee (\neg p \wedge \neg q)$	by Negation laws
	$\equiv$	$\neg p \wedge \neg q$	by Identity laws

(b) $(p \wedge q) \to (p \vee q) \equiv \mathbf{t}$.

$$
\begin{aligned}
(p \wedge q) \to (p \vee q) &\equiv \neg(p \wedge q) \vee (p \vee q) && \text{by Representation of If-Then as Or}\\
&\equiv (\neg p \vee \neg q) \vee (p \vee q) && \text{by De Morgan's laws}\\
&\equiv \neg p \vee (\neg q \vee p \vee q) && \text{by Associative laws}\\
&\equiv \neg p \vee (\neg q \vee q \vee p) && \text{by Commutative laws}\\
&\equiv \neg p \vee (\mathbf{t} \vee p) && \text{by Negation laws}\\
&\equiv \neg p \vee \mathbf{t} && \text{by Universal bound laws}\\
&\equiv \mathbf{t} && \text{by Universal bound laws}
\end{aligned}
$$

(c) $p \to (q \vee r) \equiv (p \wedge \neg q) \to r$.

$$
\begin{aligned}
p \to (q \vee r) &\equiv \neg p \vee (q \vee r) && \text{by Representation of If-Then as Or}\\
&\equiv (\neg p \vee q) \vee r && \text{by Associative laws}\\
&\equiv \neg(\neg(\neg p) \wedge \neg q) \vee r && \text{by De Morgan's laws}\\
&\equiv \neg(p \wedge \neg q) \vee r && \text{by Double negation laws}\\
&\equiv (p \wedge \neg q) \to r && \text{by Representation of Or as If-Then}
\end{aligned}
$$

(d) $p \to (q \to r) \equiv (p \wedge q) \to r$.

$$
\begin{aligned}
p \to (q \to r) &\equiv \neg p \vee (q \to r) && \text{by Representation of If-Then as Or}\\
&\equiv \neg p \vee (\neg q \vee r) && \text{by Representation of If-Then as Or}\\
&\equiv (\neg p \vee \neg q) \vee r && \text{by Associative laws}\\
&\equiv \neg(\neg(\neg p) \wedge \neg(\neg q)) \vee r && \text{by De Morgan's laws}\\
&\equiv \neg(p \wedge q) \vee r && \text{by Double negation laws}\\
&\equiv (p \wedge q) \to r && \text{by Representation of Or as If-Then}
\end{aligned}
$$