University of New Brunswick
Faculty of Computer Science

CS1303 Discrete Structures - Midterm Exam

February 26th, 2021;
Time Allowed: 50 minutes

Student Name: ____________________________ **Student No.:** _____________________

Instructions
This paper contains 5 multiple choice questions and 3 other questions, and comprises 3 pages.
Answer ALL questions. This is an open-book examination.
The marking scheme is shown in the left margin and [100] constitutes full marks.

The following table may be needed for taking this examination.

Given any statement variables p, q, and r, a tautology $\mathbf{t}$ and a contradiction $\mathbf{c}$, the following logical equivalences hold.

1.	*Commutative laws:*	$p \wedge q \equiv q \wedge p$	$p \vee q \equiv q \vee p$
2.	*Associative laws:*	$(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$	$(p \vee q) \vee r \equiv p \vee (q \vee r)$
3.	*Distributive laws:*	$p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$	$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$
4.	*Identity laws:*	$p \wedge \mathbf{t} \equiv p$	$p \vee \mathbf{c} \equiv p$
5.	*Negation laws:*	$p \vee \sim p \equiv \mathbf{t}$	$p \wedge \sim p \equiv \mathbf{c}$
6.	*Double negative law:*	$\sim(\sim p) \equiv p$	
7.	*Idempotent laws:*	$p \wedge p \equiv p$	$p \vee p \equiv p$
8.	*Universal bound laws:*	$p \vee \mathbf{t} \equiv \mathbf{t}$	$p \wedge \mathbf{c} \equiv \mathbf{c}$
9.	*De Morgan's laws:*	$\sim(p \wedge q) \equiv \sim p \vee \sim q$	$\sim(p \vee q) \equiv \sim p \wedge \sim q$
10.	*Absorption laws:*	$p \vee (p \wedge q) \equiv p$	$p \wedge (p \vee q) \equiv p$
11.	*Negations of* $\mathbf{t}$ *and* $\mathbf{c}$*:*	$\sim \mathbf{t} \equiv \mathbf{c}$	$\sim \mathbf{c} \equiv \mathbf{t}$

In addition, Representation of If-Then as Or (Implication Equivalence) means
$p \to q \equiv \neg p \vee q$, which may also be used.

[**50**] 1. **Multiple choice questions:** read each question carefully and choose the correct answer: A, B, C or D. Make sure you only choose one answer for each question.

[10] (1) Find which of the following sentences is a proposition: ________.

A. CS1303 is very easy.

B. Drink carrot juice!

C. Is there life of Mars?

D. Java language belongs to high-level programming languages, doesn't it?

[10] (2) Let $A = \{a, b, c, d\}$. If we list all the strings of length 4 over A with at least three characters that are the same into a string set B. What is the size of set B ________.

A. 16 B. 8 C. 4 D. 32

[10] (3) Which of the following statements is false? ________

A. $a \in \{a, b, c\}$ B. $\{c\} \in \{a, b, \{c\}\}$

C. $\{c\} \subseteq \{a, b, \{c\}\}$ D. $\{a\} \subseteq \{a, b, \{c\}\}$

[10] (4) Let $\oplus$ denote as the *exclusive or* operation. Which of the following compound propositions is a tautology? ________.

A. $(p \oplus p) \vee q$ B. $(\neg p \oplus \neg q) \vee p$ C. $(p \oplus \neg q) \vee p$ D. $(p \oplus \neg p) \vee q$

[10] (5) Let $P(x)$ be the statement "$x^2 > x$". What is the truth values of the quantification $\forall x\ P(x)$ where the domain consists of all positive integers? What is the truth values of the quantification $\forall x\ P(x)$ where the domain consists of all positive real numbers? ________.

A. true, true B. true, false

C. false, true D. false, false

[**20**] 2. Show that

$$p \to (q \to r) \equiv (p \wedge \neg r) \to \neg q$$

by

[10] (a) Using truth tables;

[10] (b) Using logical equivalences, and state clearly which law(s) you are using in each step.

[**20**] 3. Some of the following arguments are valid by universal modus ponens or universal modus tollens; others are invalid. State which are valid and which are invalid. Justify your answers.

[10] (a)

All healthy people eat an apple a day.
Alice is a healthy person.
$\therefore$ Alice eats an apple a day.

[10] (b)

For every student x, if x studies discrete mathematics, then x is good at logic.
Bob is good at logic.
$\therefore$ Bob studies discrete mathematics.

[**10**] 4. The logician Raymond Smullyan describes an island containing two types of people: knights who always tell the truth and knaves who always lie. Now, you are visiting the island and have the following encounters with natives.

Three natives A, B, C address you as follows:

A says: Both B and C are knights.

B says: C is a knave.

C says: B is a knave.

How many knaves are there?

END OF PAPER