University of New Brunswick
Faculty of Computer Science

CS1303 Discrete Structures - Quiz 2

March 26th, 2021;
Time Allowed: 50 miniutes

Student Name: _____ **Student No.:** _____

Instructions
This paper contains 4 questions, and comprises 1 page.
Answer ALL questions. This is an open-book examination.
The marking scheme is shown in the left margin and [100] constitutes full marks.

[**30**] 1. Determine whether the statement is true or false. Prove the statement directly from the definitions if it is true, and give a counterexample if it is false.

[10] (1) The sum of any five consecutive integers is divisible by 5.

[10] (2) For every integer $p > 2$, if p is prime then $p^2 - 1$ is divisible by 4.

[10] (3) For any integer n, $n^2 - 5$ is not divisible by 4.

[**30**] 2. For every integer n, if n^2 is even then n is even.

[15] (1) Prove the above statement by contradiction.

[15] (2) Prove the above statement by contraposition.

[**20**] 3. Prove the following statement using mathematical induction: For every integer $n \geq 2$,

$$3^2 + 3^3 + 3^4 + \cdots + 3^n = \frac{3(3^n - 3)}{2}.$$

[**20**] 4. A real number a is set, such that $a + 1 + \dfrac{1}{a+1}$ is integer. Prove that $(a+1)^n + \dfrac{1}{(a+1)^n}$ is also integer for all integer $n \geq 1$.

END OF PAPER