CS1303 Discrete Structure

Tutorial 2021-03-25

1. Let $A = \{2, 3\}$, $B = \{\varnothing, 1, \{2, 3\}\}$. Determine which of the statements given below are true and which are false:

ϕ = empty set

$\phi \subseteq B$

$\boxed{\phi \subseteq A}$

$\boxed{\forall S.\ \phi \subseteq S.}$

S set

(1) $\{1\} \subset A$. F
(2) $\{1\} \in A$. F
(3) $\varnothing \subset B$. ? — T
(4) $\varnothing \in B$. T

2. Let $A = \{\varnothing, a, \{b, c\}\}$, $B = \{b, c\}$. Determine which of the statements given below are true and which are false:

$\{a\} \in A$

$\boxed{\forall x.\ x \in B \to x \in A}$

(1) $\boxed{\{a\}} \subset A$. T
(2) $\{a\} \in A$. F
(3) $B \subset A$. F
(4) $B \in A$. T

3. Let A, B, and C be finite sets. Prove the **inclusion and exclusion formula for the three sets** A, B, and C:

$$|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |A \cap C| - |B \cap C| + |A \cap B \cap C|.$$

4. For every positive integer n, if A and $B_1, B_2, B_3, \cdots$ are any sets, then

$$A \cap \left(\bigcup_{i=1}^{n} B_i \right) = \bigcup_{i=1}^{n} (A \cap B_i)$$

5. Find a counterexample to show that the following each statement is false.

For all sets A, B, and C,

$$(A \cup B) \cap C = A \cup (B \cap C).$$

$$(A - B) \cap C = A - (B \cap C)$$

3. Let A, B, and C be finite sets. Prove the **inclusion and exclusion formula for the three sets** A, B, and C:

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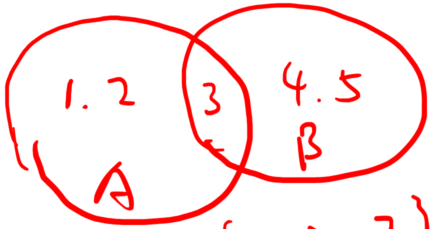

$A = \{1, 2, 3\} \Rightarrow |A| = 3$

$B = \{3, 4, 5\} \Rightarrow |B| = 3$

$A \cup B = \{1, 2, 3, 4, 5\}$

$A \cap B = \{3\}$

$|A \cup B| = 5$

$|A \cap B| = 1$

PI: $\boxed{|A \cup B| = |A| + |B| - |A \cap B|}$

$$|A \cup B \cup C| = |(A \cup B) \cup C|$$
$$= |A \cup B| + |C| - |(A \cup B) \cap C|$$
$$= |A| + |B| - |A \cap B| + |C| - |(A \cup B) \cap C|$$
$$= |A| + |B| + |C| - |A \cap B| - |A \cap C| - |B \cap C| + |A \cap B \cap C|$$

$$|(A \cup B) \cap C| = |(A \cap C) \cup (B \cap C)|$$

distributive laws

$$= |A \cap C| + |B \cap C| - |(A \cap C) \cap (B \cap C)|$$
$$= |A \cap C| + |B \cap C| - |A \cap B \cap C|$$

$\boxed{C \cap C = C}$

4. For every positive integer n, if A and $B_1, B_2, B_3, \cdots$ are any sets, then

$$A \cap \left(\bigcup_{i=1}^{n} B_i \right) = \bigcup_{i=1}^{n} (A \cap B_i)$$

$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

L R

$A \cap \bigcup_{i=1}^{n} B_i \neq \phi$

① $L \subseteq R$ ② $R \subseteq L$

①
$\forall x.\ x \in A \cap (\bigcup_{i=1}^{n} B_i)$

$\equiv \forall x.\ (x \in A) \wedge (x \in \bigcup_{i=1}^{n} B_i)$

$\equiv \forall x.\ (x \in A) \wedge (x \in B_1 \vee x \in B_2 \vee \cdots \vee x \in B_n)$

$\equiv \forall x.\ (x \in A \wedge x \in B_1) \vee (x \in A \wedge x \in B_2) \vee \cdots \vee (x \in A \wedge x \in B_n)$

$\equiv \forall x.\ (x \in A \cap B_1) \vee (x \in A \cap B_2) \vee \cdots \vee (x \in A \cap B_n)$

$\equiv \forall x\ \ x \in (A \cap B_1) \cup (A \cap B_2) \cup \cdots \cup (A \cap B_n)$

$\equiv \forall x.\ \ x \in \bigcup_{i=1}^{n} (A \cap B_i)$

a . b . c statement

$a \wedge (b \vee c) \equiv (a \wedge b) \vee (a \wedge c)$

$a \vee (b \wedge c) \equiv (a \vee b) \wedge (a \vee c)$

$a \wedge (b_1 \vee b_2 \vee b_3 \cdots \vee b_n)$

$\equiv (a \wedge b_1) \vee (a \wedge b_2) \vee (a \wedge b_3) \cdots \vee (a \wedge b_n)$

$a \vee (b_1 \wedge b_2 \wedge \cdots b_n)$

$\equiv (a \vee b_1) \wedge (a \vee b_2) \wedge \cdots \wedge (a \vee b_n)$

4. For every positive integer n, if A and $B_1, B_2, B_3, \cdots$ are any sets, then

$$A \cap \left(\bigcup_{i=1}^{n} B_i \right) = \bigcup_{i=1}^{n} (A \cap B_i)$$

② $\forall x \ . \ x \in \bigcup_{i=1}^{n} (A \cap B_i)$

$\equiv \forall x . \ (x \in A \cap B_1) \vee (x \in A \cap B_2) \vee \cdots \vee (x \in A \cap B_n)$

$\equiv \forall x \ (x \in A \wedge x \in B_1) \vee (x \in A \wedge x \in B_2) \cup \cdots \cup (x \in A \wedge x \in B_n)$

$\equiv \forall x \ (x \in A) \wedge (x \in B_1 \vee x \in B_2 \cup \cdots \vee x \in B_n)$

$\equiv \forall x \ (x \in A) \wedge (x \in (B_1 \cup B_2 \cup \cdots \cup B_n))$

$\equiv \forall x . \ x \in A \cap (B_1 \cup B_2 \cup \cdots \cup B_n)$

$\equiv \forall x \ x \in A \cap \left(\bigcup_{i=1}^{n} B_i \right)$

$L \subseteq R$. $R \subseteq L$

$\Downarrow$

5. Find a counterexample to show that the following each statement is false.

Disprove . find some counterexample

For all sets A, B, and C,

$A = \{1, 2, 3\}$

$B = \{2, 3, 4\}$

$C = \{3, 4, 5\}$

$A \cup B = \{1, 2, 3, 4\}$

$\boxed{(A \cup B) \cap C} = \{3, 4\}$

$B \cap C = \{3, 4\}$

$\boxed{A \cup (B \cap C)} = \{\cancel{3}\}$

$= \{1, 2, 3, 4\}$

$\Rightarrow (A \cup B) \cap C = A \cup (B \cap C).$ ✓

$(A - B) \cap C = A - (B \cap C)$

$A - B = \{1\}$

$(A - B) \cap C = \phi$

$A - (B \cap C) = \{1, 2\}$