
Department of Mathematics and Statistics
University of New Brunswick Fredericton
Math 1003 Intro Calculus I Winter 2021
Module 1 Test
(10% Toward Final Grade)

Due Friday 5 February 2pm in Crowdmark

Work on the test must be your own. You may use the textbook (but you don't need it). You may use the approved online resources ([Symbolab](#), [Desmos](#), [WolframAlpha](#)) to help with calculations and to check your results, **but not for having the test problems solved for you**. If you are using one of these online resources, acknowledge it. Show your work and explain clearly what you are doing in your solutions.

1. A patient is admitted to the radiology unit for a test, and is immediately (at time $t = 0$) administered 10mg of a radioactive dye (tracer). The amount of dye decays exponentially and it is known that only 6mg remains after 30 minutes (at time $t = 30$).
 - (a) (1 mark) Determine the function $Q(t) = Q_0 e^{kt}$ that represents the amount of radioactive dye remaining at t minutes.
 - (b) (2 marks) The test is not reliable if less than 4mg of dye remains. What is the patient's maximum wait time t (in minutes) before the test is performed, if the test is to be reliable? Give your answer to the nearest minute.
 - (c) (2 marks) The patient can safely be released at time $t = 95$ (in minutes) after the dye was administered (at $t = 0$). What is the amount of radioactive dye remaining at $t = 95$?

2. Let $f(x) = \frac{2x^2 - 2}{x^2 + 3x + 2}$.

Evaluate the limits in (a) and (b) (or show that the limit does not exist):

- (a) (2 marks) $\lim_{x \rightarrow -1} f(x)$.
- (b) (2 marks) $\lim_{x \rightarrow -2} f(x)$.
- (c) (1 mark) Using results in (a) and (b), find all vertical asymptotes of $f(x)$. Explain your answer, don't just list some numbers.

[more questions] \rightsquigarrow

3. The function $f(x)$ is defined for all $x > 0$, and this inequality is true for all $x > 0$:

$$\frac{4x - 1}{x} \leq f(x) \leq \frac{4x^2 + 3x}{x^2}$$

- (a) (3 marks) Use the Squeeze Theorem to evaluate $\lim_{x \rightarrow +\infty} f(x)$. Don't just give a numerical answer; explain how you are using the theorem.
- (b) (2 marks) Can you use the Squeeze Theorem to conclude anything at all about $\lim_{x \rightarrow 0^+} f(x)$, assuming it exists? Explain.

4. A meme starts trending on campus social media at time $t = 0$. The proportion of student population retweeting the meme at time $t \geq 0$ (in hours) is given by the function

$$f(t) = \begin{cases} \frac{1}{1 + 2^{-t+3}} & 0 \leq t < 4 \\ c(1 - (t - 4)^2) \cos^2(2\pi t) & 4 \leq t < 5 \\ 0 & t \geq 5 \end{cases}$$

where c is an unspecified constant ($0 \leq c \leq 1$).

- (a) (1 mark) Show that $f(t)$ is continuous at $t = 5$.
- (b) (3 marks) Assuming $f(t)$ is continuous for all $t \geq 0$, determine the parameter c .
- (c) (1 mark) Assuming $f(t)$ is continuous for all $t \geq 0$, what percentage of the student population is retweeting the meme at time $t = 4.5$?

[end of test]