Department of Mathematics and Statistics
University of New Brunswick Fredericton
Math 1003 Intro Calculus I Winter 2021
Module 3 Test
(20% Toward Final Grade)
Due Friday 19 March 2pm in Crowdmark

Work on the test must be your own. You may use the textbook (but you don't need it). You may use the approved online resources (Symbolab, Desmos, WolframAlpha) to help with calculations and to check your results, but not for having the test problems solved for you. If you are using one of these online resources, acknowledge it. Show your work and explain clearly what you are doing in your solutions.

1. Calculate the derivative of each of the functions below. Do not simplify your final answers but state which derivative rules you are using in each step.

   (a) (2 marks) $f(x) = e^{-x^2}\sin(3x)$

   (b) (1 mark) $f(x) = \arcsin(\frac{1}{x})$.

   (c) (2 marks) $f(x) = (2+\sin x)^{\cos x}$. Use logarithmic differentiation.

2. Let $f(x) = \arctan(x) - \frac{1}{2}\ln(x^2+1)$.

   (a) (2 marks) Calculate $f'(x)$. Simplify your result (you will need it in the rest of this question).

   (b) (1 mark) Determine the intervals where the function $f(x)$ is decreasing.

   (c) (1 mark) An online calculator says that $f''(x) = \dfrac{x^2-2x-1}{(1+x^2)^2}$. Prove it.

   (d) (1 mark) Determine the intervals where $f(x)$ is concave down.

[more questions] ⇝

3. You know from class that the linear approximation of a differentiable function $f(x)$ near the point $x = a$ is given by

$$L(x) = f(a) + f'(a)(x - a)\,.$$

   (a) (2 marks) Calculate the derivative of the function $f(x) = \sqrt[3]{8 + x}$.

   (b) (1 mark) Find the linear approximation $L(x)$ of $f(x) = \sqrt[3]{8 + x}$ near the point $x = 0$.

   (c) (2 marks) Use your result in (b) to approximate values of $\sqrt[3]{8.06}$ and $\sqrt[3]{7.88}$. Don't just write down some numbers; explain how you are using the linear approximation in part (b).

4. A curve in the plane is given by the equation $x^3 + y^3 = 2xy$.

   (a) (1 mark) Verify that the point $(1, 1)$ is on the curve.

   (b) (2 marks) Use implicit differentiation to express $\dfrac{dy}{dx}$ in terms of x and y.

   (c) (1 mark) Use part (b) to calculate $\dfrac{dy}{dx}$ at the point $(x, y) = (1, 1)$.

   (d) (1 mark) Write the equation of the tangent line at the point $(1, 1)$ in the form $y = mx + b$.

[end of test]