Department of Mathematics and Statistics
University of New Brunswick Fredericton
Math 1003 Intro Calculus I Winter 2021
Module 2 Test
(15% Toward Final Grade)
Due Friday 19 February 2pm in Crowdmark

Work on the test must be your own. You may use the textbook (but you don't need it). You may use the approved online resources (Symbolab, Desmos, WolframAlpha) to help with calculations and to check your results, but not for having the test problems solved for you. If you are using one of these online resources, acknowledge it. Show your work and explain clearly what you are doing in your solutions.

1. Let $f(x) = \sqrt{2x + 1}$.

   (a) (3 marks) Use the definition of derivative as a limit to find $f'(x)$.

   (b) (2 marks) Determine the equation of the tangent line to the graph of $y = f(x)$ at the point where $x = 4$.

2. A particle starts moving along the x-axis at time $t = 0$. Its position at time $t \geq 0$ is given by $x(t) = t^3 - 3t^2 + 4$. You can take for granted (no need to prove) that

$$x'(t) = 3t(t - 2)$$
$$x''(t) = 6(t - 1)$$

   (a) (1 mark) Calculate the average velocity of this particle in the time interval from $t = 0$ to $t = 3$.

   (b) (2 marks) Prove that the particle is moving to the left when $0 < t < 2$, and is moving to the right when $t > 2$.

   (c) (2 marks) Determine (with proof) the time t_0 after which the particle's velocity starts increasing.

[more questions] ⤳

3. The numbers of tundra wolves and caribou inhabiting a certain region in Northern Canada at time t are given by functions $W(t) > 0$ (wolves) and $C(t) > 0$ (caribou). Tundra wolves are predators that hunt caribou for food. A simplified mathematical model of this predator-prey interaction is given by a system of equations:

$$\frac{dC}{dt} = 0.05\,C(t) - 0.01\,C(t)W(t)$$

$$\frac{dW}{dt} = -0.05\,W(t) + 0.0001\,C(t)W(t)$$

   (a) (2 marks) You do not have to do any calculations in this part of the question. Express the following statements in terms of $\dfrac{dC}{dt}$ and $\dfrac{dW}{dt}$:

   - The number of tundra wolves in this ecosystem is increasing at time t.
   - Population sizes do not change at any time t.

   (b) (2 marks) It is expected that the number of caribou will be decreasing when there are too many wolves. Use the equations above to prove that caribou population $C(t)$ is decreasing when the number of wolves is $W(t) > 5$.

   (c) (1 mark) But if caribou population becomes too small, wolf population will be decreasing due to lack of food. Use the equations above to prove that tundra wolf population $W(t)$ is decreasing when the number of caribou is $C(t) < 500$.

4. The function $f(x)$ is defined as

$$f(x) = \begin{cases} x^2 \sin\left(\dfrac{1}{x}\right) & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$$

   (a) (2 marks) Write the limit definition of $f'(0)$ for this function. Do not calculate the limit; you'll be doing that in part (b).

   (b) (3 marks) Use the Squeeze Theorem to prove that $f(x)$ is differentiable at $x = 0$.

[end of test]